

Velocity of Light in Plantains

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ABSTRACT

Sunlight and the energy it provides are useful to the growth, development and well being of plants and animals. The velocity of light in plantains and the wavelength of the electromagnetic radiation absorbed by plantains for manufacturing its food through the processes of photosynthesis are vital for man and animal life. The velocity of light in plantains was estimated from side budded plantain pseudostem, which was inclined at a specified angle to the horizontal in Lagos Nigeria. As a result, Snell's law, Maxwell's equations and the Brewster angle computation were used to obtain the velocity of light in plantains. The property of the normal incident radiation not being refracted in the plantain pseudostem medium until reaching the plantain pseudostem-air interface was used as the main underlying assumption for this study. The results show that the dielectric permittivity of plantain pseudostem was about three times that of air. The velocity of light in plantain was also found to be a fraction of the velocity of light in vacuum. The wavelength of light for plantain obtained in this study was found to be closer to ultraviolet radiation than it is to blue light. This startling discovery reveals that the absorbed ultraviolet radiation has enough energy to break bonds as can be seen from the side bud plantain outgrowth, which was inclined at approximately 60° to the horizontal. Consequently, ultraviolet radiation facilitates photosynthesis in plantains and possibly banana. This contradicts views that blue and red lights are necessary for photosynthesis in all plants.

Keywords: blue light, dielectric permittivity, incident light, ultraviolet, wavelength

Abbreviations: E, electric field vector; EMR, electromagnetic radiation; H, magnetic field vector; UV, ultra violet

INTRODUCTION

Visible light consists of a mixture of colours, each with different wavelengths and frequencies. Sunlight contains all the colours that make up visible light (Zimmerman and Zimmerman 1993) in the form of radiant electromagnetic energy. One of the natural uses for sunlight is the process of photosynthesis in plants (Jansen 2000). Certain wavelengths of radiation are affected far more by absorption than by scattering. This is especially so for infra red and wavelengths shorter than visible light (Zimmerman and Zimmerman 1993; Jansen 2000)

The velocity of light in a vacuum is approximately $3 \times 10^8 \text{ ms}^{-1}$. Whenever electromagnetic radiation (EMR) encounters substances of different optical densities, refraction may take place. Refraction is the bending of light as it passes from one medium to another, of different optical densities. It occurs due to different velocities of light in the different media. The refractive index (n) of a medium is a measure of its optical density (Smith and Cooper 1957; Pople 1982; Warren 1982; Zimmerman and Zimmerman 1993; Jansen 2000).

People have used light to communicate for thousands of years. It was reported that light signals warned Paul Revere of the arrival of British forces at the start of the American Revolutionary wars although the light signal did not tell him the route the invaders were taking (Groneman and Feirer 1986). Fibre optics, is being used in telecommunication by means of lightwave transmission in military jet aircrafts, satellite communication, medical microscopy, and photocopyers, because they can carry enormous amounts of information (billion of words) per second and occupy less space, while also weighing much less than electrical wires (Groneman and Feirer 1986). In addition, holography which uses patterns on a photographic film can produce three-dimensional pictures of an object. Holograms are useful in business, industry, architecture, industrial designs and building

models.

Woolam (2002) used ellipsometry which is the measurement of the interaction of polarised light and materials, to specifically obtain the optical constants for TiO_2 from the ultraviolet (UV-high eV) to the infrared (IR-small eV) employing electric fields parallel and perpendicular to the plane of incidence. It was found that the real and imaginary optical constants were not independent; rather their shapes were coupled through the Kramers-Kronig consistency. Tang *et al.* (1989) used sedimentation velocity studies to examine the effect of epiphyseal cartilage link protein on aggregate size and stability. They found that the dissociation of the cartilage link-free aggregate to be a function of pH, temperature and the capacity of the link protein to stabilise aggregate against dissociation at either decreased pH or elevated temperature. Cartwright (2007) suggests that artificial materials with exotic electromagnetic properties can be designed to guide radiation around an object. Such a perfect cloak would need to make the phase velocity of light passing through its interior catch up with the light passing around it to prevent any scattering. For perfect invisibility, infinite values of permittivity and permeability would be desired of metamaterials, which is at present impossible. However, by using cylindrical cloaks with some perturbations reduced the scattering to some thin line at the centre of the cloak.

Britto Dhas *et al.* (2007) used photoacoustic techniques including X-ray diffraction to characterise L-threonine and L-prolinium tartrate, which are new organic materials grown from submerged seed solution. The L-threonine crystals showed a prominent peak at 470 nm as the fundamental wavelength of excitation in the violet region, while L-prolinium tartrate crystals were in the green region (520 nm). Galisteo-Lopez *et al.* (2007) show phase measurements on self-assembled three-dimensional photonic crystals for light propagation in which the group velocity flips from slow to negative (subliminal) values in samples of a few μm size.

Indeed, this transition showed the existence of strong anomalies in the phase delay, which take place where out-of-plane diffraction occurs and light propagates inside the sample as interfering diffracted beams. In addition, the existence of a frequency dependent transition window shows a strong range of spectral angular redistribution of emission from spontaneous internal light sources and lasing action.

Zhang *et al.* (2004) showed theoretically that the group velocity of bismuth compounds can be slowed down by means of phase coupling in the photorefractive two-wave mixing process. Indeed, this group velocity reduction arises because of a steep variation in the phase-coupling coefficient of the angular frequency difference between two coupling beams in a two-wave mixing process. Dolling *et al.* (2006) studied negative-refractive-index materials and measured both group and phase velocities by propagating a femtosecond laser pulse through a negative-index metamaterial (indium-tin-oxide) and time resolving the transmitted pulse using interferometry. While both phase and group velocities can be negative simultaneously, at other times they found negative phase velocity and positive group velocity. Nevertheless, for all sign combinations of phase and group velocities in effective metamaterials, the Poynting vector is positive: otherwise no signal would be transmitted through the sample.

Similarly, Qui *et al.* (2008) investigated the reduction in group velocity propagation due to a steep change of the refractive index of erbium-doped optical fibre using coherent population oscillation technique. Upon employing parameters optimisation, they were able to slow down the light propagation group velocity and also obtain the needed time delays. However, the optical properties of crystals are, next to X-ray diffraction and direct chemical analyses, the most reliable properties available to distinguish and identify minerals. These optical properties depend on how visible light is transmitted through the crystal, and are, therefore, dependent on crystal structure, crystal symmetry and chemical composition of the minerals (Nelson 2002). Srivastava *et al.* (2006) showed that by introducing defects into a conventional photonic band gap structure gives anomalous and unique refractive indices, zero group velocity at certain frequency and also becomes several hundred times greater than the velocity of light. These types of materials can find applications in the construction of perfect lenses, photon trapping and other optical devices. Turukhin *et al.* (2001) studied the absorption and dispersion of ultraslow group velocity of light in an optically dense Pr^{3+} doped Y_2SiO_5 crystals using Raman excited spin coherence technique. The main interest lie in the realisation that carefully controlled slow group velocity of light might allow efficient nonlinear coupling laser pulses of low energy, which can be used to create quantum enlargement between single photons without ultrahigh cavity requirements. They can also be useful in the detection of small phase shifts and those requiring large optical dispersive properties. Sapienza *et al.* (2007) suggest that transport of light in random dielectric systems is neither described accurately by group velocity nor by phase velocity. In other words, the correct velocity, which is the energy velocity, is given by the ratio of the energy flux to the energy density. It is presumed to take into account very strong delays caused by resonant scattering.

Arbab (2009) using the New Guage transformations of Maxwell's equation showed that electromagnetic fields travel with the speed of light in vacuum or free charged medium if the generalised continuity equations are satisfied. While the foregoing shows electromagnetic fields are waves, both the current density and charge density satisfy a wave equation propagating with speed of light. Similarly, Zepfenfeld (2009) used spherical coordinates of the Gauss-Laguerre contraption to model Maxwell's equations as solutions to paraxial wave equations in optics, especially for the short wavelength limit. In addition, paraxial approximations are useful for studying coherent radiation like laser beams, beams through lenses, plantains pseudostems, focus size, and phase shifts.

Biswas and Mani (2008) using the remodeled relativity theory for the velocity of light suggest that a beam of light will bend in a gravitational field exactly as a material body would if thrown horizontally with a velocity equal to that of light. That means, they treated the motion of photons under the influence of gravitational field at par with any particle having mass. Biswas and Mani (2008) are also in agreement with Unnikrishnan (2005) that the gravitational bending of light rays is the effect of variation with position of the velocity of propagation of light under the effect of gravitational field. Incidentally, Unnikrishnan used the gravitational redshift formulation and equivalence principle to the particle or ray to arrive at this additional bending of light in the gravitational field. Wayne and Staves (2003) quote Thomas Knight (1806) that: "It can scarcely have escaped the notice of the most inattentive observer of vegetation, that in whatever position a seed is placed to germinate, its radical invariably makes an effort to descend towards the centre of the earth, whilst the elongated germen takes a precisely opposite direction." Whereas the gravitropic responses of *Phycomyces* and *Physcomitrella*, differentiation of vascular tissue and polarity of cytoplasmic streaming do not correlate with the presence of sedimenting statoliths for *characean* internodal cells, it is possible that the protoplast as a whole settles in response to gravity, and the photo-gravireceptor is present in the plasma membrane at the extracellular matrix junction.

Yano *et al.* (2003) have isolated and carried out genetic and biochemical analyses of a number of shoot gravitropism mutants of *Arabidopsis* and found that one of the cognate SNAREs (Soluble N-ethylmaleimide-sensitive factor attachment protein receptor) of AtVTI11 (a protein receptor), AtVAM3 (a syntaxin), was involved in shoot gravitropism. They also suggest that the vesicle transport to the vacuole in the endodermis was important for shoot gravitropism.

Therefore, this study shall examine the velocity of light in plantains from the plantain pseudostem lying at an incline in the gravitational field, and to specifically use the Brewster angle of its side bud outgrowth as a photosensing device for analyses.

In trying to determine the velocity of light in plantains and, possibly banana, it has been shown how chlorophyll in vegetation absorbs much of the incident blue and red light for photosynthetic purposes (Smith and Cooper 1957). Indeed, Asemota (2004) related the angle of inclination of propped wind-felled plantain plants to side bud outgrowths. This work is vital to the determination of the velocity of light in plantains because, it showed how the phototropic process led to plantain wind-damage recovery mechanisms. Whilst the Asemota (2004) work only gave a succinct summary of the novel plantain shoot side bud outgrowth from staking wind-felled plantains pseudostems, the consideration of the velocity of light in plantains was neither realised then nor discussed, therein. Indeed, this study is the result of further scrutiny and reappraisal of that landmark achievement in plantain damage recovery processes, its physiological presentations and, upon viewing it from both elementary geometrical and physical optics parlance.

This research models plantain pseudostem side bud outgrowth (Asemota 2004), purely as a phototropic optical process and to quantitatively determine the composite refractive index, wavelength, and consequently, velocity of light in plantains. This approach has been adopted especially because there is now universal agreement on the speed of light in vacuum (Gibbs 1996). It is hypothesised, therefore, that the velocity of light in plantains is closer to ultra violet radiation than it is to blue light. Indeed, this ultra violet radiation is the wavelength of light used by plantains in photosynthesis.

MATERIALS AND METHODS

The main objective for initially supporting wind-felled plantain pseudostems on forked props, which later slipped to rest on fence

wall was to achieve sustained growth a priori. Secondly, it was to observe the long-term effects of the plantain pseudostem lying at an incline to the horizontal, as long as wilting has not occurred. The resulting side bud outgrowth from the long-term inclined plantain pseudostem was both surprising and intriguing. Hence, the motivation to investigate the effect of phototropic process *vis-à-vis* the velocity of light in plantains, upon side bud outgrowth using the shortest time parameter and the angle of inclination to the horizontal.

The materials used were fence-wall supported wind-felled plantains, which produced side bud outgrowths at inclination angle of about 60° to the horizontal. The 60° incline of the plantain pseudo-stem was considered as one side of a 60° equilateral prism. The incident light rays on the inclined plantain pseudo-stem were assumed normal to the plantain surface and refraction took place within the plantain pseudo-stem in the form of a light guide or light pipe (Pople 1982; Cottingham and Greenwood 1995), resulting in total internal reflection. The equilateral prism model was chosen because it has been shown that the cylindrical shape of the plantain pseudostem can be approximately represented as the prismatic surfaces of a regular hexagon (Hetnarski 2000 Hetnarski 2000; Shames and Pitaressi 2002).

Both the wave equation and Snell's law were used to determine velocity of light in plantain, relative to the velocity of light in vacuum. Experimentally, the directions of the incident, reflected and refracted rays at an interface between two optical materials lead to:

- (a) the incident, reflected, refracted rays and the normal to the surface all lie in the same plane
- (b) the angle of reflection is equal to the angle of incidence for all wavelengths and for any pair of substances, and
- (c) for monochromatic light and for given pairs of substances on opposite sides of the interface, the ratio of the sines of the angles, where both angles are measured from the normal to the surface, is equal to the inverse ratio of the two indexes of refraction (Young 1992). The above experimental results coupled with that fact that the incident and refracted rays and the normal to the surface all lie in the same plane is called the law of refraction or Snell's law after Willebrord Snell (1591-1626), Young (1992).

Wave model of electromagnetic energy

The relationship between wavelength (λ) and frequency (ν) of electromagnetic radiation is based on the following formulae, where c is the velocity of light in vacuum.

$$c = \lambda\nu \text{ (Woodward and Sheehy 1983)} \quad (1)$$

$$\nu = c/\lambda \text{ (Woodward and Sheehy 1983)} \quad (2)$$

Whenever electromagnetic radiation passes from one substance to another, both the velocity of light and wavelength change, while the frequency remains essentially the same (Morton 1971; Warren 1979).

Refraction as described by Snell's law states that for a given frequency of light, the product of the index of refraction and the sine of the angle between the rays and a line normal to the interface is constant (Smith and Cooper 1957; Morton 1971; Pople 1982; Warren 1982; Zimmerman and Zimmerman 1993; Jansen 2000). So,

$$n_p = c/c_p \text{ (Woodward and Sheehy 1983)} \quad (3)$$

where n_p is the refractive index of plantain, c is the velocity of light in vacuum and c_p is the velocity of light in plantain.

Since, light is an electromagnetic radiation (EMR), it is expected to obey the laws of reflection and refraction, which apply to plane polarised waves (Morton 1971; Jackson 2001). Consider plane-polarised waves making oblique incidence with plantain pseudo-stem (dielectric), for ease of modelling. Dielectrics are different from plasmas and conducting media, because dielectrics do not contain free charges but tightly bound charges to form atoms and molecules. Upon application of external electric field, dielectrics form small separations of bound charges like electrostatic dipoles, called polarisation (Jordan and Balmain 1998). As a result,

Maxwell (1998) states that the theory of residual discharge, absorption of electricity, electrification, or polarisation, deserves a careful investigation, and will probably lead to important discoveries relating to the internal structure of bodies.

If light waves are horizontally polarised, the E-field vector is parallel to the boundary (**Fig. 2**), and vertically polarised if the H-field vector is parallel to the boundary (**Fig. 2**). For both conditions, the tangential component of E must be continuous across the boundary for points on the Z-axis, where this axis is taken to correspond with the boundary. This implies that (Morton 1971; Jackson 2001), the velocity with which the wave front (which is perpendicular to the direction of propagation) cuts the Z-axis is the same for the incident, reflected and refracted waves. This is called the phase velocity (u_{ph}), in the Z-direction.

Thus,

$$u_{ph} = u/\sin \psi_i = u/\sin \psi_r = u_t/\sin \psi_t \text{ (Morton 1971)} \quad (4)$$

where ψ_t is the angle of refraction. Hence, $\psi_i = \psi_r$, and

$$\frac{u}{u_t} = \frac{\sin \psi_i}{\sin \psi_t} \text{ (Morton 1971)} \quad (5a)$$

But,

$$u_a = 1/\sqrt{(\mu_a \epsilon_a)} \text{ (Morton 1971)} \quad (5b)$$

$$u_p = 1/\sqrt{(\mu_p \epsilon_p)} \quad (5c)$$

where u_a and u_p in equations (5b) and (5c) are the velocities of light in air and plantain, respectively.

For simplicity and most nonmagnetic materials, the relative permeability, μ_r is very close to 1 at optical frequencies (Boast *et al.* 2000). Hence, assume $\mu_a = \mu_p = \mu_o$, so that

$$\frac{\sin \psi_i}{\sin \psi_t} = \frac{\sqrt{(\epsilon_p)}}{\sqrt{(\epsilon_a)}} = \frac{\sqrt{(\epsilon_{pr})}}{\sqrt{(\epsilon_{ar})}} \text{ (Morton 1971)} \quad (6)$$

where ϵ_{pr} and ϵ_{ar} are relative permittivities of plantain and air, respectively.

The magnitudes of reflected and refracted components are different for the horizontal and vertical polarisation. These could be determined by considering the normal components of the waves. For the horizontal polarised wave, the wave component in the x-direction for E and H are; E and, $H \cos \psi$.

Hence, the wave impedances for this component are:

$$Z_{wax} = E_i/H_i \cos \psi_i = Z_{wa}/\cos \psi_i \text{ (Morton 1971) (for the incident wave)} \quad (7)$$

$$Z_{wpx} = E_t/H_t \cos \psi_t = Z_{wp}/\cos \psi_t \text{ (Morton 1971) (for the transmitted wave)} \quad (8)$$

Since $E_i/H_i = Z_{wa}$, and $E_t/H_t = Z_{wp}$, then:

$$E_t = 2Z_{wpx}E_i/(Z_{wpx} + Z_{wax}) \text{ (Morton 1971)} \quad (9)$$

and,

$$E_r = (Z_{wpx} - Z_{wax})E_i/(Z_{wpx} + Z_{wax}) \text{ (Morton 1971)} \quad (10)$$

There will be no reflected wave if $Z_{wpx} = Z_{wax}$, so

$$Z_{wa}/\cos \psi_i = Z_{wp}/\cos \psi_t = Z_{wp}/\sqrt{(1 - \sin^2 \psi_t \epsilon_{ar}/\epsilon_{pr})} \text{ (Morton 1971) (from equation 6)} \quad (11a)$$

Assuming, $\mu_a = \mu_p = \mu_o$ (as before)

$$Z_{wa} = \sqrt{(\mu_o/\epsilon_a)}, \text{ (Morton 1971)} \quad (11b)$$

and

$$Z_{wp} = \sqrt{(\mu_o/\epsilon_p)} \text{ (Morton 1971)} \quad (11c)$$

So, for equality,

$$1/(\sqrt{\epsilon_a})\cos \psi_i = 1/(\sqrt{\epsilon_p})(\sqrt{1 - \sin^2 \psi_i \epsilon_{ar}/\epsilon_{pr}}) \quad (12)$$

Hence,

$$\epsilon_a \cos^2 \psi_i = \epsilon_p (1 - \sin^2 \psi_i \epsilon_{ar}/\epsilon_{pr}) \quad (13)$$

or,

$$1 - \sin^2 \psi_i = \epsilon_{pr}/\epsilon_{ar} - \sin^2 \psi_i \quad (14)$$

There is no value of ψ_i for which this equation can be true, except where $\epsilon_{pr} = \epsilon_{ar}$, and hence for all angles of incidence, there will be both a transmitted and a reflected wave from the horizontally polarised wave (Morton 1971; Jackson 2001).

For the vertically polarised wave (Fig. 2), the components of E and H for the wave component in the x-direction are E cos ψ and H. Thus, the wave impedances for this x-component are:

$$Z_{wax} = E_i \cos \psi_i / H_i = Z_{wa} \cos \psi_i \quad (15)$$

and,

$$Z_{wpx} = E_t \cos \psi_t / H_t = Z_{wp} \cos \psi_t \quad (16)$$

Equations (15) and (16), again give the transmitted and reflected waves.

In this case, there will be no reflection at an angle of incidence ψ'_i , which makes

$$Z_{wax} = Z_{wpx} \quad (17)$$

(That is) if:

$$\cos \psi'_i (\sqrt{(\mu_o/\epsilon_a)}) = \sqrt{(\mu_o/\epsilon_p)} (\sqrt{1 - \sin^2 \psi'_i \epsilon_{ar}/\epsilon_{pr}}) \quad (18)$$

or, if:

$$1 - \sin^2 \psi'_i = (\epsilon_{ar}/\epsilon_{pr}) (1 - \sin^2 \psi'_i \epsilon_{ar}/\epsilon_{pr}) \quad (19)$$

The above yields,

$$1 - \sin \psi'_i = (\sqrt{(\epsilon_{ar}/\epsilon_{pr} - 1)})/\sqrt{(\epsilon_{ar}^2/\epsilon_{pr}^2 - 1)} = \sqrt{(\epsilon_{pr}/(\epsilon_{pr} + \epsilon_{ar}))} \quad (20)$$

Equation (20) represents a real angle and it is called the Brewster angle (Morton 1971; Woodward and Sheehy 1983). This is also the angle for which inclined plantain (60°) pseudo-stem produces a side bud outgrowth on the incline (Asemota 2004).

At this particular angle of incidence of a vertically polarised wave, all the energy of the incident wave is transmitted through the plantain pseudo-stem (Morton 1971; Woodward and Sheehy 1983; Asemota 2004).

Upon substituting values for $\sin 60^\circ$ into equation (20), becomes:

$$(\sqrt{3})/2 = \sqrt{(\epsilon_{pr}/(\epsilon_{pr} + \epsilon_{ar}))}$$

$$3/4 = (\epsilon_{pr}/(\epsilon_{pr} + \epsilon_{ar}))$$

$$\text{Therefore, } 3\epsilon_{ar} = \epsilon_{pr} \quad (21)$$

The relative composite permittivity of plantain is about three times that of air.

$$\text{Let } \mu_o = 4\pi \times 10^{-7} \text{ H/m}$$

$$\epsilon_o = 8.854 \times 10^{-12} \text{ F/m (Svoboda 2004)}$$

Upon substituting the above values into equation (5c), gives:

$$u_p = 1/\sqrt{(\mu_p \epsilon_p)} = 1/\sqrt{(3\pi \times 8.854 \times 10^{-12} \times 4 \times 10^{-7})} \quad (22)$$

$$u_p = 173,078,092.10 \text{ ms}^{-1}$$

where, u_p is the velocity of light in plantains.

Let $c = 299792458 \text{ ms}^{-1}$ (Gibbs 1996).

From equation (4), it is pertinent to determine the composite (because plantain pseudostem contains vacuolar spaces, liquid and fibre) refractive index of plantains as:

$$n_p = 299792458 \text{ ms}^{-1} / 173078092.10 \text{ ms}^{-1} \quad (23)$$

$$n_p = 1.732122502.$$

Sellmeier equation

To confirm the composite refractive index of plantains through another method, the Sellmeier equation (Dyakov *et al.* 1988; Jundt 1997; Shen *et al.* 2004; Kolev *et al.* 2005; Deng *et al.* 2006; Kim and Sarangan 2007; Paul *et al.* 2007; Wen-Le *et al.* 2008; Wikipedia 2008a; Barboza and Cudney 2009) was used because it is an empirical relationship between refractive index and wavelength for a particular transparent medium. Sellmeier equations are useful because they fairly accurately describe refractive index in a wide wavelength range. There are also a great variety of modified Sellmeier formulae to include frequency and temperature dependence of refractive indices (Paschotta 2008). Thus the simple form of Sellmeier equation becomes,

$$n^2(\lambda) = 1 + B_1\lambda^2/(\lambda^2 - C_1) + B_2\lambda^2/(\lambda^2 - C_2) + B_3\lambda^2/(\lambda^2 - C_3) \quad (24)$$

where $B_{1,2,3}$ and $C_{1,2,3}$ are the experimentally determined Sellmeier coefficients. These coefficients are quoted for λ in μm . This λ is the vacuum wavelength and not the material wavelength itself, which is $\lambda/n(\lambda)$.

From equation (24), the general Sellmeier equation is of the form:

$$n^2(\lambda) = 1 + \sum_i (B_i\lambda^2/(\lambda^2 - C_i)) \quad (25)$$

with each term of the sum representing an absorption resonance of strength B_i at a wavelength $\sqrt{C_i}$ (Kolev *et al.* 2005).

Kolev *et al.* (2005) used stoichiometric lithium tantalate (SLT, LiTaO_3) for wavelengths near the mid-infra red absorption edge, because its lower susceptibility to photo-refractive damage makes it more suitable for high-power applications. Whereas stoichiometric lithium tantalate has the advantage of a higher transparency in the 5-6 μm range, a large deviation in the mid-infra red absorption edge was also found in that region. Jundt (1997) fitted measured tuning data for an optical parametric oscillator to determine the extraordinary index of congruent lithium niobate. He found that phase-matching predictions were accurate for temperatures between ambient and 250°C and wavelengths between 0.4 and 5 μm . Deng *et al.* (2006) used the continuous tunable mid-infra red radiation for the extraordinary index of periodically poled LiNbO_3 crystal to improve upon the Sellmeier equation, by employing difference-frequency-generation and quasi-phase-matching techniques. This improved equation was suitable for temperatures between 25 and 180°C and mid-infra red wavelengths ranging from 2.8 to 4.8 μm . Kim and Sarangan (2007) derived a Sellmeier equation for $\text{Al}_x\text{Ga}_{1-x}\text{As}$ material by measuring the refractive index of GaAs and AlAs, separately with temperature dependent resonant cavity structure. While the equation was applicable in the 1460-1580 nm range and between 26 and 86°C, these could be extrapolated to other wavelength and temperature ranges. The Sellmeier equation for the refractive index dispersion of congruently grown LiTaO_3 , accurate from 0.3 to 5 μm and from 23 to above 200°C was studied. The resulting equation accurately predicted tuning curves for optical parametric generation in the infra red as well as the correct quasi-phase-matching conditions for frequency conversion into the ultraviolet using second-harmonic generation (Barboza and Cudney 2009). Paul *et al.* (2006) describe a Sellmeier equation and the predicted extraordinary refractive indices for 5% MgO:CLN (congruent lithium niobate) in the mid-infra red spectral range from 1.3 to 5 μm , and for temperatures from 40 to 200°C. This material is used in nonlinear optical processes for

generating radiation in the visible, infra red and THz regimes. Although various applications demand certain fixed wavelengths, accurate wavelength stability can be achieved by knowing the temperature sensitivity of the system.

If all the terms are specified for a material at long wavelengths far from the absorption peaks, the value of n tends to (Wikipedia 2008a):

$$n \approx \sqrt{1 + \sum_i B_i} \approx \sqrt{\epsilon_r} \text{ (Paschotta 2008)} \quad (26)$$

where ϵ_r is the relative dielectric constant of plantain.

Upon substituting for $\epsilon_p = \epsilon_r$ in equations (26) and (21), gives:

$$\begin{aligned} n_p &\approx \sqrt{\epsilon_p} \approx \sqrt{\epsilon_r} \approx \sqrt{3} \\ n_p &\approx 1.732050808 \end{aligned} \quad (27)$$

which is in close agreement with equation (23) to three places of decimal.

From equation (2), the wavelength of light radiation passing through plantains can be determined as follows:

The velocity of light in plantain is less than the velocity of light in vacuum or air. If f is the frequency of the wave and $\tau = 1/f$ is the period, the time interval between successive crests passing a fixed point in space (Electron9.phys.utk.edu); then becomes,

$$\lambda_1 = v_1\tau = c\tau/n_1, \text{ and } \lambda_2 = v_2\tau = c\tau/n_2, \text{ or}$$

$$\lambda_1/\lambda_2 = n_2/n_1 \quad (28)$$

Therefore,

$$\lambda_1 n_1 = \lambda_2 n_2 \quad (29)$$

where λ_1 is the wavelength of light in the first medium (vacuum or air) and n_1 is the refractive index in the first medium (vacuum or air); and λ_2 is the wavelength of light in the second medium (plantain).

Using the absolute refractive index for air under standard conditions (1.0002918) for light having the D-line of sodium (589.3×10^{-9} m) (Subrahmanyam *et al.* 2004).

From equation (29), becomes

$$\lambda_p = \lambda_0 n_0 / n_p \quad (30)$$

where λ_p is the wavelength of plantain, λ_0 is the wavelength of light wave in vacuum and n_0 is the refractive index of light wave in vacuum.

Upon substituting the above values into equation (30), gives

$$\lambda_p = 589.3 \times 10^{-9} \text{ m} / 1.732122502 \quad (31)$$

$$\lambda_p \approx 340.2 \times 10^{-9} \text{ m} \quad (32)$$

RESULTS AND DISCUSSION

Maxwell made the velocity of light, c , central to electromagnetic theory and James Bradley calculated the speed of light by deducing that the starlight falling on the earth should appear to come from a slight angle. Incidentally, the Bradley calculated the speed of light to be about 298,000 km/s, and it is close to the currently accepted value (Biswas and Mani 2008). By considering the propagation of light through a material, it was observed that group velocities could lie either below or well above the speed of light, c , in vacuum (Galisteo-Lopez *et al.* 2007). Hence, slow-light and superluminal light propagations are common. Superluminal behaviour represents negative group velocity, where it appears the pulse peak exits the medium before even entering it. While this behaviour is believed to arise from anomalous dispersion, a number of media such as bulk absorptive media, reflective dielectric multilayers, transparent media presenting gain, diffraction gratings with sub wavelength apertures, metamaterials, or serial loop structures

operating on the MHz, have been known to exhibit such behaviours in the visible and infra red ranges of the electromagnetic spectrum (Turukhin *et al.* 2001; Galisteo-Lopez *et al.* 2007).

Turukhin *et al.* (2001) demonstrated an ultraslow group velocity of light of about 45 m/s in a Pr^{3+} doped Y_2SiO_5 rare-earth solid crystal, which was limited by the inhomogeneous broadening of the ground state transition. From the foregoing, it can be seen that although doping and measurement of new materials provide interesting phase and group velocity characteristics, the angle measurement technique of light rays as reported by Biswas and Mani (2008) on the work of James Bradley (1728), after about 300 years is close to the accepted speed of light value, today. The above statement should lend credence to the velocity of light in plantains using the Brewster's angle measurement method.

A common method for the direct measurement of the refractive index of a material uses the deflection of light beam by a prism, which is composed of the material of interest, (plantain). Additionally, every optical process that is sensitive to the refractive index might be suitable. So, Paul *et al.* (2007) used the temperature-dependent Sellmeier equation to study the refractive index of 5 mol % MgO doped congruent lithium niobate, and concluded that the wavelength and temperature ranges considered provided a quantitatively reliable Sellmeier equation. Martin Britto Dhas *et al.* (2007) show that photoacoustic spectroscopy is advantageous to measure the thermal diffusivity, thermal conductivity, thermal effusivity and thermal expansion of two new materials, because it is both a non-invasive and non-destructive testing technique. Additionally, the photoacoustic spectrum of a material is proportional to the optical absorbing power of that material for any wavelength. This above factor was used to measure the energy gap of L-threonine and L-proline tartrate crystals. In sum, it can be seen that the most accurate method for studying the velocity of light in plantains would be to use the angle subtended on the inclined plantain and using a prism to approximate the sides of plantain pseudostem (as a hexagon). Although the shoot outgrowth results in the tearing of the plantain pseudostem, as a consequence of the wavelength of light needed for photosynthesis, the incident sunlight on the inclined plantain pseudostem represents both non-invasive and non-destructive testing techniques. In fact, the side shoot outgrowth is advantageous because it represents an adaptive recovery process for hitherto; uprooted, mangled, damaged and wind-thrown plantain. However, the discussion of the velocity of light in plantains, which have been approximated using six-sided hexagonal prisms, follows.

Plantain pseudostem presenting about 50° to sunlight has its rays displaced 135° at total internal reflection inside the plantain for the sunrays to appear, overhead (**Fig. 3**). For plantain lying about 57° to the horizontal, it also has its rays displaced about 123° (**Fig. 4**). At about 60° inclination angle, the displacement was about 120° . The angle of refraction (or reflection), which is 60° tallies with the Brewster's angle, and it is also the most probable angle for which plantain pseudostems bud outgrowth occurs (Asemota 2004) (**Figs. 1, 5**). This 60° angle equally coheres with the inclination angles at which roof structures of greenhouses are built (Kut and Hare 1983), in order to optimise maximum sun angle normal to inclined plantain pseudostem.

In **Fig. 6**, the normal and parallel incident sunlight are displaced within the plant about 135° for plantains lying about 45° to the horizontal. Although plantain shoot outgrowth only occurred for this angle of inclination with good rains (Asemota 2004), the plantain outgrowth was visible on the upright pseudostem about 83.8 cm above the ground. While the damaged upper part was supported about 91.4 cm above the ground, termites later ate the roots during a prolonged drought to wither and fall under its own weight.

Although, plantain lying about 40° to the horizontal had about 140° displacement angle, empirical evidence (**Figs. 1, 7**) shows that plantains lying at about 40° to the horizontal withered and died without side bud outgrowths (Asemota

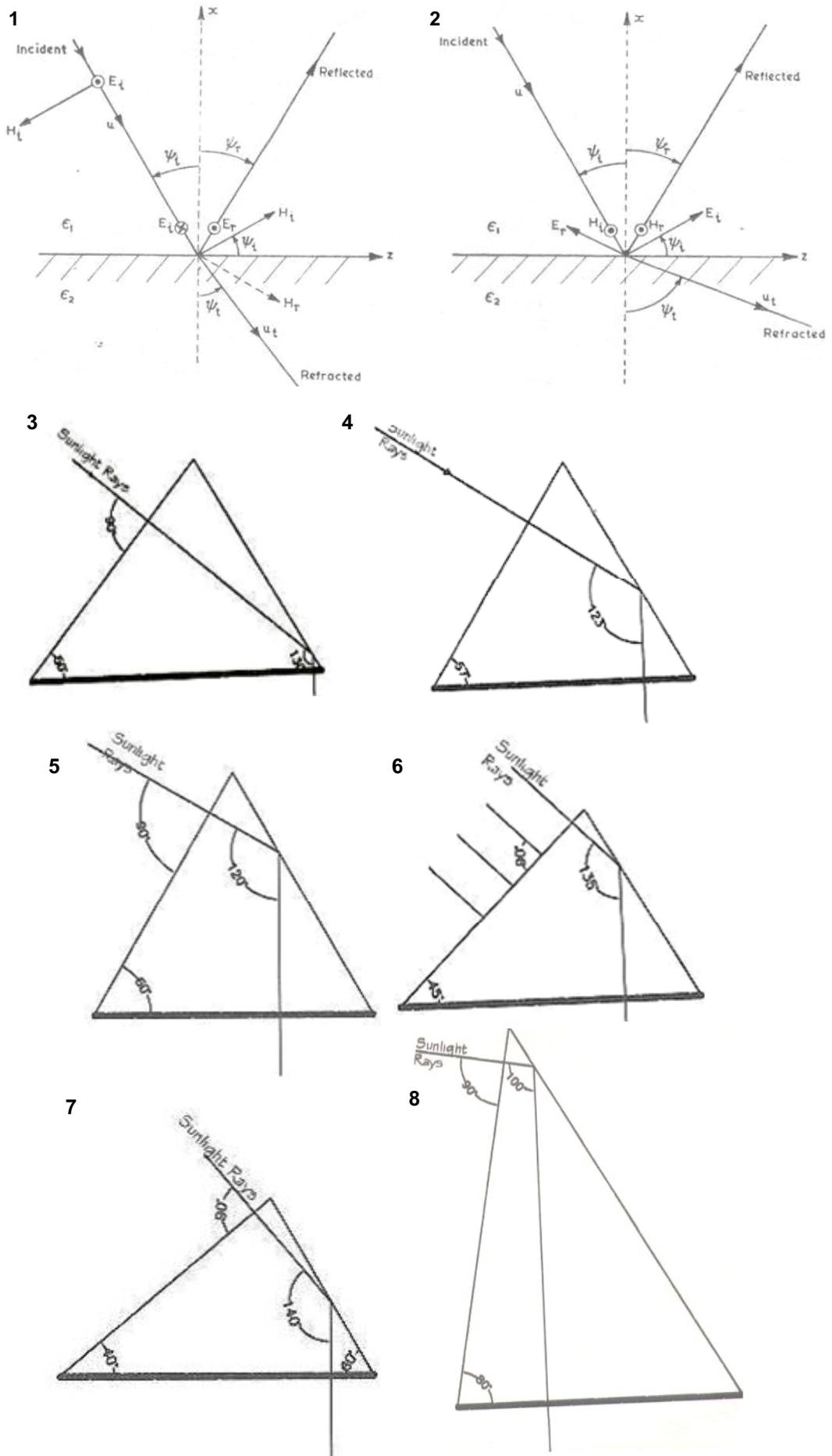


Fig. 1 Wind-felled plantain responding to sunlight in phototropism. Fig. 2 Incident, reflected and refracted plane polarised waves in plantain. Figs. 3, 4, 5, 6, 7, 8: 50°, 57°, 60°, 45°, 40° and 80° sunlight incidence angle on plantain pseudostem, respectively.

2004). Also, the base diameter to plantain height (length) ratio does not seem to support the geometrical optic description as shown in Fig. 7. This is so because, the plantain pseudostem diameter/height ratio of about (0.08) does not seem to tally with the already established maximum plantain diameter of about 30 cm across (Cobley and Steele 1976), for which the highest height is about 3.65 m (Crane and Balerdi 2006). The diameter/height ratio in this study ($8.5/8.0 = 1.0625$) was about 1.0625. In other words, the height of plantain pseudostem is about 94% its diameter. This situation of the diameter being much bigger than the height of plantains or banana for light to be refracted inside the pseudostem is both physically and naturally, not feasible. So, it can be inferred that side bud outgrowth is not likely to occur for angles equal to or less than 40° to the horizontal.

Conversely, for plantains lying about 80° to the horizontal (Asemota 2004), there was no outgrowth but a lengthening of the pseudostem after a dormancy of about 18 months. Also, plantain pseudostem standing erect at about 90° (Fig. 8) constitutes the natural and usual pattern through which, sunlight gets into the plantain pseudostem via the leaves (Wilkins 1989), as determined by plantains and banana adaptation to their niche habitats.

The composite refractive index (or index of refraction) is a measure for how much the velocity of light (or other waves) is reduced inside the plantain. Two common properties are directly related to plantain refractive index. First, light rays change direction when they cross the interface between air and plantain pseudostem. Second, light reflects partially from surfaces that have a refractive index different from that of their surroundings (Woodward and Sheehy 1983; Wikipedia 2008b).

In real materials like plantains and bananas, the polarisation does not respond instantaneously to an applied field. This causes dielectric loss, which can be expressed by a permeability that is both complex and frequency (wavelength) dependent (Rensselaer Polytechnic Institute and Wagner 1999; Wikipedia 2008a, 2008b).

Since the refractive index of a material like plantain varies with frequency (and thus wavelength) of light, it is usual to specify the corresponding vacuum wavelength at which the refractive index is measured. Generally, this is done at variously well-defined spectral emission lines. For instance, n_D is the refractive index at the Fraunhofer "D" line, the centre of the yellow sodium double emission at 589.29 nm wavelength (Rensselaer Polytechnic Institute and Wagner 1999; Subrahmanyam *et al.* 2004; Wikipedia 2008a, 2008b).

Additionally, the refractive index of a material like plantain is the most important property of any optical system that uses refraction. Since refraction is a fundamental physical property of a substance, it is often used to identify a particular substance, confirm its purity or measure its concentration. Refractive index is used to measure the chemical composition of solids (glasses and gemstones), liquids and gases (Nelson 2002). Refractive index is most commonly used to measure the concentration of solute in an aqueous solution and possible metal oxides contained in the material. The refractive index, which is a material's optical density, can also be used to determine the sugar content for a solution of sugar (Rensselaer Polytechnic Institute and Wagner 1999; Subrahmanyam *et al.* 2004; Wikipedia 2008a, 2008b).

The wavelength of light determined for plantains in this study was found to be closer to ultraviolet radiation than to blue light. Ultraviolet radiation is important to the understanding of the formation and function of side bud outgrowths, phototropic processes and photosynthesis in plantains. This is so because the energy of ultraviolet (UV) radiation is sufficient to break bonds in some molecules (Ophardt 1998; Larcher 2003).

Ultraviolet radiation (0.27–0.4 μm), has a higher quantum energy content than any other waveband of solar radiation and it is capable of driving a range of photo-chemical reactions requiring very high energy (Woodward and Sheehy 1983).

The wavelength (0.34 μm) deduced in this study for plantains also agrees with, and corroborates the already established fact that in the visible or photosynthetically action region (PAR) (0.4–0.7 μm), the leaf absorbs 90% of the radiation incident upon it (Woodward and Sheehy 1983).

Use was made of the Rayleigh scattering, such that the intensity of emitted radiation is inversely proportional to the fourth power of wavelength of the radiation. The wavelengths of blue, red and calculated UV lights were approximately 0.4, 0.7 and 0.3 μm , respectively. The intensity of the emitted blue light for plantains is proportional to $1/(0.4)^4 = 39$. The intensity of the emitted red light for plantains is proportional to $1/(0.7)^4 = 4.2$, and intensity of the calculated emitted UV light in plantains is proportional to $1/(0.3)^4 = 123$.

Thus, the intensity ratio of the emitted blue light to red light was $39/4.2 = 9$ or about 9 times more intense than that of the red light in plantains. The intensity ratio of the emitted UV light to blue light was $123/39 = 3$ or about 3 times more intense than that of the blue light in plantains. The intensity ratio of computed UV light to red light was $123/4.2 = 29$ or about 29 times more intense than that of the red light in plantains. These intensity ratios tend to corroborate why plantains and possibly bananas, which are long day plants require ultraviolet radiation for their form and function, especially for their food manufacturing processes.

CONCLUSION

Although the velocity of light in plantains gives an indication of how fast light travels within the pseudostem, the influence of light in plants is rather complicated. While, it has been shown that light drives the photochemistry and affects the shape and size of the plant, the amounts of assimilate utilisation is also dependent on the quality and velocity of light, which ultimately affect the rates of photosynthesis (Woodward and Sheehy 1983). Additionally, phytochrome, which acts as a photoreceptor plays vital photoperiodic roles, because large changes in light quality affect the spectral absorption characteristics in leaves. Consequent changes in irradiance periods, which is a function of the velocity and wavelength of light undoubtedly influence plantain's development and its reproductive systems.

The velocity of light in plantain was also found to be a fraction of the velocity of light in vacuum ($u_p = 173,078,092.10 \text{ ms}^{-1}$).

The wavelength of light for plantain obtained in this study was found to be closer to ultraviolet radiation than it is to blue light ($\lambda_p \approx 340.2 \times 10^{-9} \text{ m}$). This startling discovery reveals that the absorbed ultraviolet radiation had enough energy to break bonds as can be seen from the side bud plantain outgrowth, which was inclined at approximately 60° to the horizontal. This assertion is especially so because the intensity ratio of UV radiation to that of red light is about 30 times stronger than red light. Consequently, ultraviolet radiation facilitates photosynthesis in plantains and possibly bananas. This discovery and major contribution to plantains and banana biology, contradicts the earlier long held views that blue and red lights were necessary for photosynthesis in all plants. As and thus could be a boom to the pharmaceutical industry.

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